

Five Dimensional Non-Supersymmetric Black Holes and Strings

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Abstract

We study non-supersymmetric solutions of five dimensional $N = 2$ supergravity theories coupled to an arbitrary number of abelian vector multiplets. The solutions constructed can be considered as deformations of known supersymmetric black hole and string solutions. General constraints coming from the analysis of the equations of motion are derived. These represent explicit conditions on the charges of the black holes and strings. The constraints are analyzed for theories where the scalar manifolds are symmetric spaces and explicit solutions are constructed in cases where the prepotential of the theory factorizes into a linear and a quadratic term.

1 Introduction

In recent years great effort has been devoted towards the study and classification of supersymmetric solutions of supergravity theories. In particular, in five dimensions, it has been possible to find a complete classification of supersymmetric solutions with various fractions of supersymmetry [1]. The research in this domain is motivated by the fact that supersymmetric gravitational solutions play an important role in our understanding of the microscopic origin of entropy, stringy duality symmetries and the conjectured AdS/CFT correspondence. Though much is known about the structure of solutions preserving fractions of supersymmetry, the same can not be said about solutions breaking all of the supersymmetries. In this paper we are mainly interested in finding non-rotating non-supersymmetric black hole and string solutions in five dimensional supergravity coupled to abelian vector multiplets. Non supersymmetric solutions for these theories were first considered in [2] where an explicit solution was found for the so-called *STU* model with three independent electric charges. Our present work can be considered as an elaboration and an extension of the results found in [2]. We will find non-supersymmetric electrically charged black holes for the gauged and ungauged theories. We will also find magnetically charged string solutions in the ungauged theories.

We organize our paper as follows. Section 2 contains a summary of the basic notions of $N = 2$, $D = 5$ supergravity and very special geometry which will be used in our subsequent analysis. In section 3 non-supersymmetric solutions are derived for all models of gauged and ungauged $N = 2$, $D = 5$ supergravity models, both the scalars and the gauge fields are expressed in terms of harmonic functions and the equations of motion are reduced to one constraint on these harmonic functions. Section 4 contains a similar analysis for the non-supersymmetric black string solutions of the ungauged theories. Section 5 contains a discussion on models with symmetric scalar manifolds where the constraints simplify and an explicit analysis is given for solutions where the prepotential factorizes into a linear and a quadratic term. The known solutions of the so called *STU* model are discussed within our analysis. Section 6 summarizes our results.

2 $N = 2$ Supergravity theory

Here we review some of the basics of the theories of five dimensional $N = 2$ supergravity coupled to abelian vectormultiplets. Such theories were first constructed in [3] to which the reader can be referred for detailed discussion. The bosonic action of the theory in terms of the so-called very special geometry was later given in [4]. A large class of the $N = 2$, $D = 5$ models are obtained from the compactification of eleven dimensional supergravity, the low energy limit of M-theory, on a Calabi-Yau threefold [5]. The bosonic action of the $N = 2$ ungauged supergravity coupled to

abelian vector multiplets can be written as

$$S = \frac{1}{16\pi G} \int \left(R \star 1 - G_{IJ} (F^I \wedge *F^J + dX^I \wedge *dX^J) - \frac{1}{6} C_{IJK} F^I \wedge F^J \wedge A^K \right) \quad (2.1)$$

where $F^I = dA^I$, where A^I are the 1-forms representing the n Abelian gauge fields. In our analysis our metric has signature $(-, +, +, +, +)$. The scalars X^I are constrained by the condition

$$\mathcal{V}(X) = \frac{1}{6} C_{IJK} X^I X^J X^K = X_I X^I = 1. \quad (2.2)$$

and thus can be regarded as being functions of $n - 1$ unconstrained scalars ϕ^i . The coupling G_{IJ} depends on the scalars via

$$G_{IJ} = \frac{9}{2} X_I X_J - \frac{1}{2} C_{IJK} X^K. \quad (2.3)$$

Contracting G_{IJ} with X^J and $\partial_i X^J$, we arrive at the following useful equations

$$G_{IJ} X^J = \frac{3}{2} X_I, \quad G_{IJ} \partial_i X^J = -\frac{3}{2} \partial_i X_I. \quad (2.4)$$

where $\partial_i = \frac{\partial}{\partial \phi^i}$. The bosonic part of the action of the corresponding $U(1)$ -gauged supergravity is given by (2.1) with an additional potential term $\chi^2 \mathcal{U}$, where the scalar potential \mathcal{U} can be written as

$$\mathcal{U} = 9V_I V_J (X^I X^J - \frac{1}{2} G^{IJ}) \quad (2.5)$$

where V_I are constants. The scalar equations in the $U(1)$ -gauged theory can be written as

$$\begin{aligned} & -\nabla^\alpha \nabla_\alpha X_I + \left(\frac{1}{6} C_{MNI} - \frac{1}{2} X_I C_{MNI} X^J \right) \nabla_\alpha X^M \nabla^\alpha X^N \\ & - \frac{1}{2} \left(X_M X^P C_{NPI} - \frac{1}{6} C_{MNI} - 6X_I X_M X_N + \frac{1}{6} X_I C_{MNI} X^J \right) F^M{}_{\beta_1 \beta_2} F^{N\beta_1 \beta_2} \\ & - 3\chi^2 V_M V_N \left(\frac{1}{2} G^{ML} G^{NP} C_{LPI} + X_I (G^{MN} - 2X^M X^N) \right) = 0. \end{aligned} \quad (2.6)$$

The Einstein equations are

$$R_{\mu\nu} = G_{IJ} \left(F^I{}_{\mu\lambda} F^J{}_{\nu}{}^\lambda + \nabla_\mu X^I \nabla_\nu X^J - \frac{1}{6} g_{\mu\nu} F^I{}_{\rho\sigma} F^{J\rho\sigma} \right) - \frac{2}{3} \chi^2 g_{\mu\nu} \mathcal{U} \quad (2.7)$$

and the Maxwell gauge equations are given by

$$d(G_{IJ} \star F^J) = -\frac{1}{4} C_{IJK} F^J \wedge F^K. \quad (2.8)$$

3 Non-Supersymmetric Black Holes

In this section we construct a class of non-supersymmetric black holes in both the gauged and the ungauged supergravity theories, we consider the gauged theory first. We take the following ansatz for the metric:

$$ds^2 = -e^{-4A} f dt^2 + e^{2A} \left(\frac{dr^2}{f} + r^2 d\Omega_{3,k}^2 \right) \quad (3.1)$$

where $A = A(r)$, $f = f(r)$ and

$$d\Omega_{3,k}^2 = \begin{cases} d\xi^2 + \sin^2 \xi (d\theta^2 + \sin^2 \theta d\phi^2) & k = 1 \\ d\xi^2 + \xi^2 (d\theta^2 + \sin^2 \theta d\phi^2) & k = 0 \\ d\xi^2 + \sinh^2 \xi (d\theta^2 + \sin^2 \theta d\phi^2) & k = -1 \end{cases} \quad (3.2)$$

corresponds to the metric on \mathbf{S}^3 , \mathbf{R}^3 or \mathbf{H}^3 according as $k = 1$, $k = 0$ or $k = -1$.

The non-vanishing Ricci tensor components are given by

$$\begin{aligned} R_{tt} &= -\frac{f}{2r} e^{-6A} (4A'' r f + 4f' A' r - r f'' + 12A' f - 3f'), \\ R_{rr} &= -\left(A'' + 6A'^2 + 3\frac{A'}{r} \right) - \frac{1}{2rf} (-4f' A' r + r f'' + 3f'), \\ R_{\xi\xi} &= -f (3A' r + A'' r^2 + 2) - f' (A' r^2 + r) + 2k \end{aligned} \quad (3.3)$$

and

$$R_{\theta\theta} = \begin{cases} \sin^2 \xi R_{\xi\xi} & k = 1 \\ \xi^2 R_{\xi\xi} & k = 0 \\ \sinh^2 \xi R_{\xi\xi} & k = -1 \end{cases} \quad (3.4)$$

$$R_{\phi\phi} = \begin{cases} \sin^2 \xi \sin^2 \theta R_{\xi\xi} & k = 1 \\ \xi^2 \sin^2 \theta R_{\xi\xi} & k = 0 \\ \sinh^2 \xi \sin^2 \theta R_{\xi\xi} & k = -1 \end{cases} \quad (3.5)$$

The prime denotes differentiation with respect to the radial coordinate r .

To proceed, we assume that the only non-vanishing component of the gauge field strengths is $F^I_{tr} = F^I_{tr}(r)$, and that the scalars X^I depend only on r . We take the following as an ansatz for the gauge fields ¹

$$F^I_{rt} = \frac{1}{2} e^{-4A} G^{IJ} \partial_r \tilde{H}_J, \quad \text{for } k = 0, 1, \quad (3.6)$$

$$F^I_{rt} = \frac{i}{2} e^{-4A} G^{IJ} \partial_r \tilde{H}_J, \quad \text{for } k = -1, \quad (3.7)$$

¹Note that in the case $k = -1$, we have complexified the gauge field strengths. Such solutions, strictly speaking, are not non-extremal solutions of the standard $N = 2$ supergravity. Rather, they are solutions of a modified theory, with a sign change in the Maxwell term in the action.

where \tilde{H}_I constitute a set of harmonic functions $\tilde{H}_I = \tilde{h}_I + \frac{\tilde{q}_I}{r^2}$. Both the gauge field equations and the Bianchi identities hold without further constraint.

Next consider the Einstein equations; these are equivalent to

$$f'' + \frac{7}{r}f' + \frac{8}{r^2}f - \frac{8k}{r^2} = -36\chi^2 e^{2A} V_I V_J \left(\frac{1}{2} G^{IJ} - X^I X^J \right) \quad (3.8)$$

and

$$G_{IJ} F_{rt}^I F_{rt}^J = e^{-4U} \left(-3f' A' - 3f A'' + \frac{f''}{2} - \frac{9f A'}{r} + \frac{f'}{2r} + \frac{2(k-f)}{r^2} \right) \quad (3.9)$$

and

$$G_{IJ} \partial_r X^I \partial_r X^J = - \left(3A'' + \frac{9}{r} A' + 6A'^2 \right). \quad (3.10)$$

To satisfy these constraints, we adopt the same ansatz for the scalars as for the ungauged supersymmetric black hole solutions [6]:

$$X_I = \frac{1}{3} e^{-2A} H_I(r). \quad (3.11)$$

where $H_I(r)$ are harmonic functions. This constraint is sufficient to ensure that (3.10) is satisfied.

We set

$$H_I = \delta V_I + \frac{q_I}{r^2} \quad (3.12)$$

where δ is a non-zero constant. Then (3.8) can be rewritten as

$$f'' + \frac{7}{r}f' + \frac{8}{r^2}f - \frac{8k}{r^2} = \frac{9\chi^2}{\delta^2} ((r^2 e^{6A})'' + \frac{7}{r} (r^2 e^{6A})' + 8e^{6A}) \quad (3.13)$$

This equation has

$$f = k - \frac{\mu}{r^2} + \frac{9\chi^2}{\delta^2} r^2 e^{6A} \quad (3.14)$$

as a solution. The remaining condition (3.9) from the Einstein equations is then equivalent to

$$G^{IJ} S_{IJ} = 0 \quad (3.15)$$

where

$$\begin{aligned} S_{IJ} &= (\tilde{q}_I \tilde{q}_J - k q_I q_J) - \frac{1}{2} \mu \delta (q_I V_J + q_J V_I), \quad \text{for } k = 0, 1, \\ S_{IJ} &= -(\tilde{q}_I \tilde{q}_J - q_I q_J) - \frac{1}{2} \mu \delta (q_I V_J + q_J V_I), \quad \text{for } k = -1, \end{aligned} \quad (3.16)$$

Lastly, we consider the scalar equation (2.6). It is straightforward but tedious to show that constraint (3.15) and the scalar equations are equivalent to the constraint

$$C_{MNI} G^{ML} G^{NT} S_{LT} + 8X^M S_{MI} - 12X_I X^M X^N S_{MN} = 0. \quad (3.17)$$

We note that if we contract (3.17) with X^I , then equation (3.15) is obtained

For supersymmetric black holes with event horizon topology S^3 , we take $\mu = 0$, $k = 1$ and $H_I = \tilde{H}_I$, and (3.17) is satisfied with $S_{IJ} = 0$. However, for the deformed solutions with $\mu \neq 0$, if $S_{IJ} = 0$ for all I, J then it is straightforward to show that there must exist constants α, β such that

$$\tilde{q}_I = \alpha q_I + \beta V_I \quad (3.18)$$

and furthermore q_I and V_I must be linearly dependent. In order to find solutions for which the charges are not so strongly constrained, instead of solving $S_{IJ} = 0$ for all I, J , one must solve the weaker condition given by (3.17).

Finally, the non-supersymmetric black hole solutions of the ungauged theory are obtained by setting $\chi = 0$ and $k = 1$ throughout the gauged solution. One minor subtlety is that for the gauged solutions, the asymptotic values of the harmonic functions H_I given in (3.12) are fixed (up to an overall scale) in terms of the constants V_I which appear in the construction of the theory. However, for the ungauged solutions, the constants V_I appearing in (3.12) are arbitrary. Setting $\mu = 0$, one recovers the supersymmetric black hole solutions presented in [6, 7].

4 Non-Supersymmetric Magnetic Strings

In this section we construct non-supersymmetric black string solutions of the ungauged theory. The metric is given by

$$ds^2 = e^{-2B} (-f dt^2 + dz^2) + e^{4B} \left(\frac{1}{f} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) \quad (4.1)$$

where $B = B(r)$, $f = f(r)$. The non-vanishing components of the Ricci tensor are given by

$$\begin{aligned} R_{tt} &= -e^{-6B} f \left(f' B' - \frac{f'}{r} - \frac{f''}{2} \right) - e^{-6B} f^2 \left(B'' + \frac{2B'}{r} \right), \\ R_{zz} &= e^{-6B} f' B' + f e^{-6B} \left(B'' + \frac{2B'}{r} \right), \\ R_{rr} &= - \left(2B'' + 6B'^2 - \frac{f' B'}{f} + \frac{f''}{2f} + \frac{4B'}{r} + \frac{f'}{rf} \right), \\ R_{\theta\theta} &= -f (4B' r + 2B'' r^2 + 1) - r f' (2B' r + 1) + 1, \\ R_{\phi\phi} &= R_{\theta\theta} \sin^2 \theta. \end{aligned} \quad (4.2)$$

We also assume that the scalars X^I depend only on r , and that the only non-zero components of the gauge field strengths are given by

$$F_{\theta\phi}^I = \alpha^I \sin \theta \quad (4.3)$$

for constant α^I . With these choices, the gauge field equations and Bianchi identities hold without further constraint. The Einstein equations (taking $\chi = 0$) then fix

$$f = 1 - \frac{\mu}{r} \quad (4.4)$$

together with the constraints

$$\frac{e^{-4B}}{r^4} G_{IJ} \alpha^I \alpha^J = -3 \left(f' B' + f B'' + \frac{2B' f}{r} \right), \quad (4.5)$$

$$G_{IJ} \partial_r X^J \partial_r X^I = -3 \left(B'' + 2B'^2 + \frac{2B'}{r} \right). \quad (4.6)$$

To satisfy the constraint (4.6) we set

$$X^I = e^{-2B} H^I \quad (4.7)$$

where

$$H^I = h^I + \frac{q^I}{r} \quad (4.8)$$

are harmonic functions. Then (4.5) can be rewritten as

$$G_{IJ} U^{IJ} = 0 \quad (4.9)$$

where

$$U^{IJ} = \alpha^I \alpha^J - q^I q^J - \frac{1}{2} \mu (h^I q^J + h^J q^I). \quad (4.10)$$

Finally, consider the scalar equations (2.6) (with $\chi = 0$). It is straightforward to show that the scalar equations, together with (4.9) are equivalent to

$$\left(X_M X^P C_{NPI} - \frac{1}{6} C_{MNI} - \frac{9}{2} X_I X_M X_N \right) U^{MN} = 0. \quad (4.11)$$

Again if we contract (4.11) with X^I the condition (4.9) is obtained. Note that (4.11) can be rewritten entirely in terms of the harmonic functions H^I as

$$U^{MN} \left(C_{MM_1 M_2} C_{IN M_3} C_{M_4 M_5 M_6} - \frac{1}{6} C_{MNI} C_{M_1 M_2 M_3} C_{M_4 M_5 M_6} \right. \\ \left. - \frac{3}{4} C_{IM_1 M_2} C_{MM_3 M_4} C_{NM_5 M_6} \right) H^{M_1} H^{M_2} H^{M_3} H^{M_4} H^{M_5} H^{M_6} = 0. \quad (4.12)$$

Clearly, one way to satisfy (4.12) is to set $U^{MN} = 0$ for all M, N ; however just as in the case of the black holes, this constraint is too restrictive on the charges. Finally, for $\mu = 0$, one obtains the supersymmetric magnetic strings constructed in [8]

5 Explicit Solutions

In this section, we shall construct solutions for the models related to Jordan algebras, i.e., models where the scalar manifold is a symmetric space. These theories were first constructed by Gunaydin, Sierra and Townsend [3] where it was shown that \mathcal{V} are in one-to-one correspondence with the norm forms of Euclidean (formally real) Jordan algebras J of degree 3. The target spaces take the form

$$\mathcal{M} = \frac{\text{Str}_0(J)}{\text{Aut}(J)}. \quad (5.1)$$

Here $\text{Str}_0(J)$ denotes the invariance group of the norm (reduced structure group) of the Jordan algebra J and $\text{Aut}(J)$ is its automorphism group. Non-simple Jordan algebras of degree three are of the form $\mathbb{R} \oplus \Gamma_n$, where Γ_n is the Jordan algebra associated with a quadratic form. The corresponding symmetric scalar manifolds are

$$\mathcal{M} = SO(1, 1) \times \frac{SO(n-1, 1)}{SO(n-1)}. \quad (5.2)$$

In this case, $\mathcal{V}(X)$ is factorizable into a linear times a quadratic form in $(n-1)$ scalars, which for the positivity of the kinetic terms in the Lagrangian, must have a Minkowski metric. For Simple Euclidean Jordan algebras $\mathfrak{h}_3(\mathbb{A})$ generated by 3×3 Hermitian matrices over the four division algebras $\mathbb{A} = \mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$, the corresponding spaces \mathcal{M} are, respectively:

$$\mathcal{M} = \frac{\text{SL}(3, \mathbb{R})}{\text{SO}(3)}, \quad \frac{\text{SL}(3, \mathbb{C})}{\text{SU}(3)}, \quad \frac{\text{SU}^*(6)}{\text{USp}(6)}, \quad \frac{\text{E}_{6(-26)}}{\text{F}_4}.$$

For the simple Jordan algebras [9], an element for the four families $\mathfrak{h}_3^{\mathbb{A}}$ can be written in the form

$$L = \begin{pmatrix} \alpha & z^* & y^* \\ z & \beta & x \\ y & x^* & \gamma \end{pmatrix} \quad (5.3)$$

where $(\alpha, \beta, \gamma) \in \mathbb{R}$ and $(x, y, z) \in \mathbb{A}$. The cubic norm \mathcal{V} is given by

$$\mathcal{V} = \det L = \alpha\beta\gamma - (\alpha|x|^2 + \beta|y|^2 + \gamma|z|^2) + 2\text{Re}(xyz) \quad (5.4)$$

In all of these cases, the following constraints hold:

$$C^{IJK} = \delta^{II'} \delta^{JJ'} \delta^{KK'} C_{I'J'K'}, \quad (5.5)$$

$$C_{IJK} C_{J'(LM} C_{PQ)K'} \delta^{JJ'} \delta^{KK'} = \frac{4}{3} \delta_{I(L} C_{MPQ)}, \quad (5.6)$$

$$X^I = \frac{9}{2} C^{IJK} X_J X_K \quad (5.7)$$

$$G^{IJ} = 2X^I X^J - 6C^{IJK} X_K \quad (5.8)$$

In the case of the black hole solutions, the constraint (3.17) can then be rewritten as

$$(C^{IMN} - 6X^M C^{IJN} X_J + X^I X^M X^N) S_{MN} = 0 \quad (5.9)$$

or equivalently

$$S_{MN} \left(\frac{1}{36} C^{M_1 M_2 M_3} C^{M_4 M_5 M_6} C^{LMN} - \frac{1}{6} C^{M_1 M_2 M_3} C^{NLM_4} C^{MM_5 M_6} + \frac{1}{8} C^{LM_1 M_2} C^{MM_3 M_4} C^{NM_5 M_6} \right) H_{M_1} H_{M_2} H_{M_3} H_{M_4} H_{M_5} H_{M_6} = 0. \quad (5.10)$$

Observe that this equation is identical (up to a trivial raising and lowering of indices) to that found for the black string solutions in (4.12). This is also expected because of the duality symmetry discussed in [10]. Hence, it suffices to solve the equation (5.10), (or equivalently (5.9)). These equations can be simplified slightly to give

$$\left(\frac{1}{2} C_{IM_1 M_2} C^{M_1 M N} C^{M_2 N_1 N_2} - \delta_I^M C^{NN_1 N_2} \right) S_{MN} H_{N_1} H_{N_2} = 0. \quad (5.11)$$

In principle, the constraints on the charges can be obtained by expanding this equation in powers of r ; however these constraints are highly non-linear and in general they do not appear tractable.

To proceed, we consider the case when the pre-potential \mathcal{V} factorizes into a linear times a quadratic form as

$$\mathcal{V} = \frac{1}{2} X^1 (\eta_{ab} X^a X^b), \quad a, b = 2, \dots, n \quad (5.12)$$

and η_{ab} is a Minkowski metric on $\mathbb{R}^{1, n-2}$.

Then we note the useful identities

$$\begin{aligned} \eta_{ab} X^b &= 9X_a X_1, \quad X^1 = \frac{9}{2} \eta^{ab} X_a X_b, \quad X^a = 9X_1 \eta^{ab} X_b \\ X^1 X_1 &= \frac{1}{3}, \quad X^a X_a = \frac{2}{3}, \quad \eta^{ab} X_b = \frac{1}{3} X^1 X^a. \end{aligned} \quad (5.13)$$

It is then straightforward to show that the constraints (5.9) give the two conditions

$$S_{11} = 0, \quad (5.14)$$

and

$$2X^c \eta^{ab} S_{bc} - X^a \eta^{bc} S_{bc} = 0. \quad (5.15)$$

Note that the components S_{1a} are not constrained by (5.9). The constraint $S_{11} = 0$ is equivalent to

$$\begin{aligned} \tilde{q}_1 &= \sqrt{\mu \delta q_1 V_1}, \quad \text{for } k = 0, \\ \tilde{q}_1 &= \sqrt{q_1^2 + \mu \delta q_1 V_1}, \quad \text{for } k = 1, \\ \tilde{q}_1 &= \sqrt{q_1^2 - \mu \delta q_1 V_1}, \quad \text{for } k = -1, \end{aligned} \quad (5.16)$$

and (5.15) is equivalent to

$$\left(\delta V_d + \frac{q_d}{r^2}\right) (2\eta^{cd}\eta^{ab} - \eta^{ad}\eta^{bc}) S_{bc} = 0. \quad (5.17)$$

This gives two equations (for $k = 0, 1$)

$$\begin{aligned} 2V^b \tilde{q}_b \tilde{q}_a - (2kV^b q_b + \mu\delta V^b V_b) q_a - (\tilde{q}^b \tilde{q}_b - kq^b q_b) V_a &= 0 \\ 2q^b \tilde{q}_b \tilde{q}_a - (\tilde{q}^b \tilde{q}_b + kq^b q_b) q_a - \mu\delta q^b q_b V_a &= 0 \end{aligned} \quad (5.18)$$

where $q^a = \eta^{ab} q_b$, $V^a = \eta^{ab} V_b$, $\tilde{q}^a = \eta^{ab} \tilde{q}_b$.

There are a number of cases to consider. In the first case, there exist λ, σ such that

$$\tilde{q}_a = \lambda q_a + \sigma V_a, \quad (5.19)$$

then (5.18) can be rewritten as

$$\begin{aligned} \left(2(\lambda^2 - k)V^b q_b + (2\lambda\sigma - \mu\delta)V^b V_b\right) q_a + \left(\sigma^2 V^b V_b - (\lambda^2 - k)q^b q_b\right) V_a &= 0 \\ \left((\lambda^2 - k)q^b q_b - \sigma^2 V^b V_b\right) q_a + \left((2\lambda\sigma - \mu\delta)q^b q_b + 2\sigma^2 V^b q_b\right) V_a &= 0 \end{aligned} \quad (5.20)$$

There are then two sub-cases.

(i) $\sigma^2 V_b V^b - (\lambda^2 - k)q^b q_b \neq 0$. Then there exists θ such that $q_a = \theta V_a$ for all a , where

$$\begin{aligned} \theta \neq 0, \quad V^a V_a \neq 0, \quad \theta^2(\lambda^2 - k) - \sigma^2 &\neq 0 \\ \theta^2(\lambda^2 - k) + \theta(2\lambda\sigma - \mu\delta) + \sigma^2 &= 0 \end{aligned} \quad (5.21)$$

(ii) $\sigma^2 V_b V^b - (\lambda^2 - k)q^b q_b = 0$. There are then four possibilities:

1. $q_a = 0$ for all a with $\sigma = 0$ and $V^b V_b \neq 0$.
2. $V_a = 0$ for all a with $\lambda^2 = k$, $2\lambda\sigma - \mu\delta \neq 0$, $q^b q_b \neq 0$.
3. $\sigma \neq 0$ with

$$q^b V_b = -\frac{(2\lambda\sigma - \mu\delta)}{2\sigma^2} q^b q_b, \quad V^b V_b = \frac{(\lambda^2 - k)}{\sigma^2} q^b q_b. \quad (5.22)$$

4. $\sigma = 0$ with $q^b q_b = 0$ and $V^b V_b = \frac{2(\lambda^2 - k)}{\mu\delta} V^b q_b$.

One can also consider the case where $V^b \tilde{q}_b = q^b \tilde{q}_b = 0$. There are then two sub-cases:

1. If $q^b q_b \neq 0$ then there exists λ such that $V_a = \lambda q_a$ for all a . λ is then fixed by $(k + \lambda\mu\delta)q^b q_b + \tilde{q}^b \tilde{q}_b = 0$,
2. $q_a = 0$ for all a , and $\tilde{q}^a \tilde{q}_a = 0$. (Note that we cannot have both $q_a = 0$ and $V_a = 0$ for all a , as this would imply $X_a = 0$ for all a , in contradiction to the constraint $X^a X_a = \frac{2}{3}$).

Similarly for the case of $k = -1$, then equation (5.15) gives two equations

$$\begin{aligned} 2V^b \tilde{q}_b \tilde{q}_a + (-2V^b q_b + \mu\delta V^b V_b)q_a - (\tilde{q}^b \tilde{q}_b - q^b q_b)V_a &= 0 \\ 2q^b \tilde{q}_b \tilde{q}_a + (-\tilde{q}^b \tilde{q}_b - q^b q_b)q_a + \mu\delta q^b q_b V_a &= 0 \end{aligned} \quad (5.23)$$

and similarly there are a number of cases to consider. Again one can consider the case when

$$\tilde{q}_a = \lambda q_a + \sigma V_a, \quad (5.24)$$

This gives

$$\begin{aligned} \left(2(\lambda^2 - 1)V^b q_b + (\mu\delta + 2\sigma\lambda)V^b V_b \right) q_a - \left((\lambda^2 - 1)q^b q_b - \sigma^2 V^b V_b \right) V_a &= 0 \\ \left((\lambda^2 - 1)q^b q_b - \sigma^2 V^b V_b \right) q_a + \left(2\sigma^2 q^b V_b + (2\lambda\sigma + \mu\delta)q^b q_b \right) V_a &= 0 \end{aligned} \quad (5.25)$$

and as for the $k = 0, 1$, we consider two sub-cases.

(i) $(\lambda^2 - 1)q^b q_b - \sigma^2 V^b V_b \neq 0$. Then there exists θ such that $q_a = \theta V_a$ for all a , where

$$\begin{aligned} \theta \neq 0, \quad V^a V_a \neq 0, \quad \theta^2(\lambda^2 - 1) - \sigma^2 &\neq 0 \\ \theta^2(\lambda^2 - 1) + \theta(2\lambda\sigma + \mu\delta) + \sigma^2 &= 0 \end{aligned}$$

(ii) $(\lambda^2 - 1)q^b q_b - \sigma^2 V^b V_b = 0$. Then we have four possibilities:

1. $q_a = 0$ for all a with $\sigma = 0$ and $V^b V_b \neq 0$.
2. $V_a = 0$ for all a with $\lambda^2 = 1$, $2\lambda\sigma + \mu\delta \neq 0$, $q^b q_b \neq 0$.
3. $\sigma \neq 0$ with

$$q^b V_b = -\frac{(2\lambda\sigma + \mu\delta)}{2\sigma^2} q^b q_b, \quad V^b V_b = \frac{(\lambda^2 - 1)}{\sigma^2} q^b q_b. \quad (5.26)$$

4. $\sigma = 0$ with $q^b q_b = 0$ and $V^b V_b = -\frac{2(\lambda^2 - 1)}{\mu\delta} V^b q_b$.

Also we consider the case when $V^b \tilde{q}_b = q^b \tilde{q}_b = 0$. Then there are two sub-cases:

1. If $q^b q_b \neq 0$ then there exists λ such that $V_a = \lambda q_a$ for all a . λ is then fixed by $\tilde{q}^b \tilde{q}_b + (1 - \lambda\mu\delta)q^b q_b = 0$,

2. $q_a = 0$ for all a , and $\tilde{q}^a \tilde{q}_a = 0$.

It is instructive to see where the non-extremal STU black hole solutions of [2] fit into this scheme. For the STU model, we take $X^1 = S$, $X^2 = T$, $X^3 = U$ with

$$\eta_{ab} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (5.27)$$

The black hole solutions with spherical horizons correspond to setting $\chi = g$, $k = 1$, $\delta = 3$ with $V_1 = V_2 = V_3 = \frac{1}{3}$ and

$$\tilde{q}_a = \mu \sinh \beta_a \cosh \beta_a, \quad q_a = \mu \sinh^2 \beta_a, \quad a = 2, 3 \quad (5.28)$$

This solution corresponds to the case (5.22) with where $\sigma \neq 0$ and $k = 1$. Consider the case for which $\beta_2 \neq \beta_3$, one finds that

$$\tilde{q}_a = \lambda q_a + \sigma V_a$$

for $a = 2, 3$, with

$$\begin{aligned} \lambda &= \frac{\sinh \beta_2 \cosh \beta_2 - \sinh \beta_3 \cosh \beta_3}{\sinh^2 \beta_2 - \sinh^2 \beta_3}, \\ \sigma &= 3\mu \left(\frac{\sinh \beta_2 \sinh \beta_3}{\sinh^2 \beta_2 - \sinh^2 \beta_3} \right) (\sinh \beta_2 \cosh \beta_3 - \sinh \beta_3 \cosh \beta_2). \end{aligned} \quad (5.29)$$

For $k = 0$, we have

$$\tilde{q}_a = \mu \sinh \beta_a, \quad q_a = \mu \sinh^2 \beta_a, \quad (5.30)$$

In this case it is easy to verify that

$$\tilde{q}_a = \lambda q_a + \sigma V_a$$

for $a = 2, 3$, with

$$\lambda = \frac{1}{(\sinh \beta_2 + \sinh \beta_3)}, \quad \sigma = 3\mu \frac{\sinh \beta_3 \sinh \beta_2}{\sinh \beta_2 + \sinh \beta_3}. \quad (5.31)$$

This belongs to the class of solutions satisfying (5.22) with $k = 0$.

For $k = -1$, we have

$$\tilde{q}_I = -\mu \sinh \beta_I \cosh \beta_I, \quad q_I = -\mu \sinh^2 \beta_I. \quad (5.32)$$

Then

$$\tilde{q}_a = \lambda q_a + \sigma V_a$$

for $a = 2, 3$, with

$$\begin{aligned} \lambda &= \frac{\sinh \beta_2 \cosh \beta_2 - \sinh \beta_3 \cosh \beta_3}{\sinh^2 \beta_2 - \sinh^2 \beta_3} \\ \sigma &= - \left(\frac{3\mu \sinh \beta_2 \sinh \beta_3}{\sinh^2 \beta_2 - \sinh^2 \beta_3} \right) (\sinh \beta_2 \cosh \beta_3 - \sinh \beta_3 \cosh \beta_2) \end{aligned} \quad (5.33)$$

This belongs to the class of solutions satisfying (5.26).

6 Discussion

We have constructed non-supersymmetric solutions of five dimensional $N = 2$ supergravity theories coupled to an arbitrary number of abelian vector multiplets. The solutions constructed are deformations of known supersymmetric black hole and string solutions. The scalar fields have the same solution as in the supersymmetric cases. However, one has to solve extra conditions involving the various charges and the parameter μ . These conditions are given for the black holes and black strings, respectively by (3.17) and (4.11). However, for supergravity models with scalars living on symmetric spaces the condition (3.17) take a much simpler form given in (5.9) which can also be obtained from (4.11) using the duality transformation discussed in [10]. We have studied the condition (5.9) for models where the prepotential factorizes into a linear and a quadratic form and derived various conditions for the existence of explicit solutions. It is of interest to find more general non-supersymmetric solutions as deformations of known general supersymmetric ones. Our results can be generalized to other supergravity models and in particular to those in four dimensions. We hope to report on this in a future publication.

Acknowledgement:

This work of W. S is supported in part by the National Science Foundation under grant number PHY-0703017.

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